Epp Theorem 1.1.1 Logical Equivalences

Given any statement variables P, Q, and R, a tautology \mathbf{t} , and a contradiction \mathbf{c} , the following logical equivalences hold.

(1) Commutative laws:

$$P \wedge Q \equiv Q \wedge P$$

 $P \vee Q \equiv Q \vee P$

(2) Associative laws:

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

 $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

(3) Distributive laws:

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

(4) Identity laws:

$$P \wedge \mathbf{t} \equiv P$$
$$P \vee \mathbf{c} \equiv P$$

(5) Negation laws:

$$P \lor \neg P \equiv \mathbf{t}$$
$$P \land \neg P \equiv \mathbf{c}$$

(6) Double negation law:

$$\neg(\neg P) \equiv P$$

(7) Idempotent laws:

$$P \wedge P \equiv P$$
$$P \vee P \equiv P$$

(8) Universal bound laws:

$$P \lor \mathbf{t} \equiv \mathbf{t}$$

 $P \land \mathbf{c} \equiv \mathbf{c}$

(9) DeMorgan's laws:

$$\neg(P \land Q) \equiv \neg P \lor \neg Q$$

$$\neg(P \lor Q) \equiv \neg P \land \neg Q$$

(10) Absorption laws:

$$P \lor (P \land Q) \equiv P$$

 $P \land (P \lor Q) \equiv P$

(11) Negations of \mathbf{t} and \mathbf{c} :

$$\begin{array}{ccc}
 \neg \mathbf{t} & \equiv & \neg \mathbf{c} \\
 \mathbf{c} & \equiv & \mathbf{t}
 \end{array}$$